Math 250 – Sect.3.3 – Increasing and Decreasing Functions and the First Derivative

I. INCREASING/DECREASING FUNCTIONS

Definition of increasing/decreasing: *A picture is worth a thousand words!!*

Theorem: Let f be a function that is continuous on the closed interval [a, b] and differentiable on the open interval (a, b).

1. If ____________________ for all x in (a, b), then f is ____________________ on [a, b].
2. If ____________________ for all x in (a, b), then f is ____________________ on [a, b].
3. If ____________________ for all x in (a, b), then f is ____________________ on [a, b].

Theorem: *The First Derivative Test*. Let c be a **critical number** of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from ______________ to ________________ at c, then f has a relative ______________________ at $x = c$.
2. If $f'(x)$ changes from ______________ to ________________ at c, then f has a relative ______________________ at $x = c$.

* In order to determine whether $f'(x)$ is greater than or less than 0, first find the **critical** numbers. Then do a *number line* test.

-example- Consider the function $f(x) = x^3 - 6.5x^2 - 38x + 240$. Find the intervals on which the function is increasing and decreasing, and the locations of any relative extrema.

ANSWER: *f* is INCREASING on ______________________________

f is DECREASING on _______________________

f has a relative MAXIMUM at _____________________

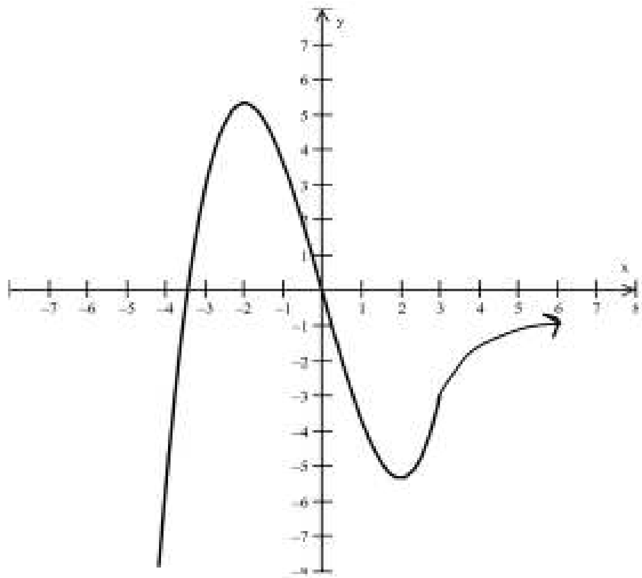
f has a relative MINIMUM at ______________________

-example- Consider the function $f(x) = (x-3)^{1/3}$. Find the intervals on which the function is increasing and decreasing, and the locations of any relative extrema.

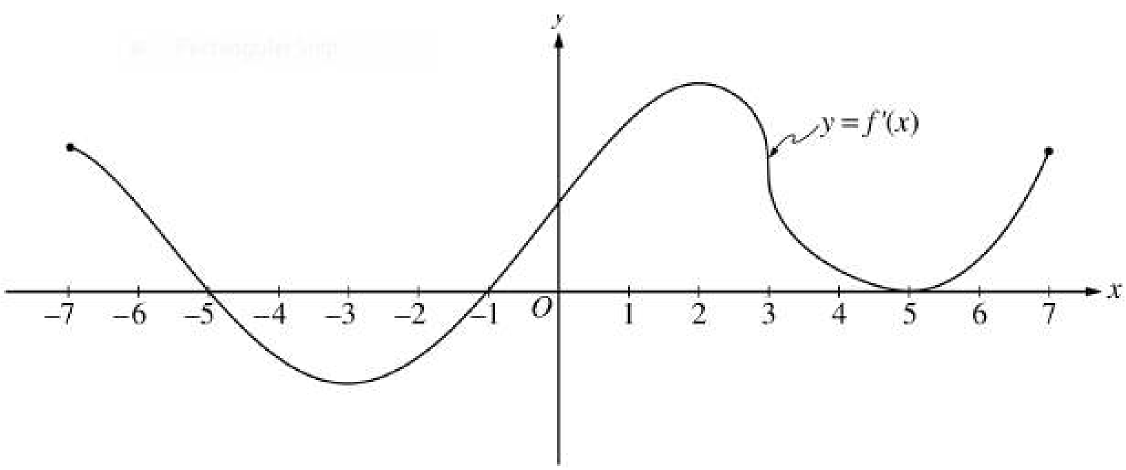
-example- Consider the function $f(x)=\dfrac{x+4}{x^2}$. Find the intervals on which the function is increasing and decreasing, and the locations of any relative extrema.

-example- Consider the function $f(x)=x+2\sin x$ on the interval $[0,\ 2\pi]$. Find the intervals on which the function is increasing and decreasing, and the locations of any relative **and absolute** extrema.

-example- Below is a graph of a function f. On the same graph, SKETCH the graph of the derivative, f'.



-example- Below is the graph of a derivative function, f'



On what intervals is f INCREASING? ____________________DECREASING?____________

Where does f have relative maximums?________________ Relative minimums?_____________